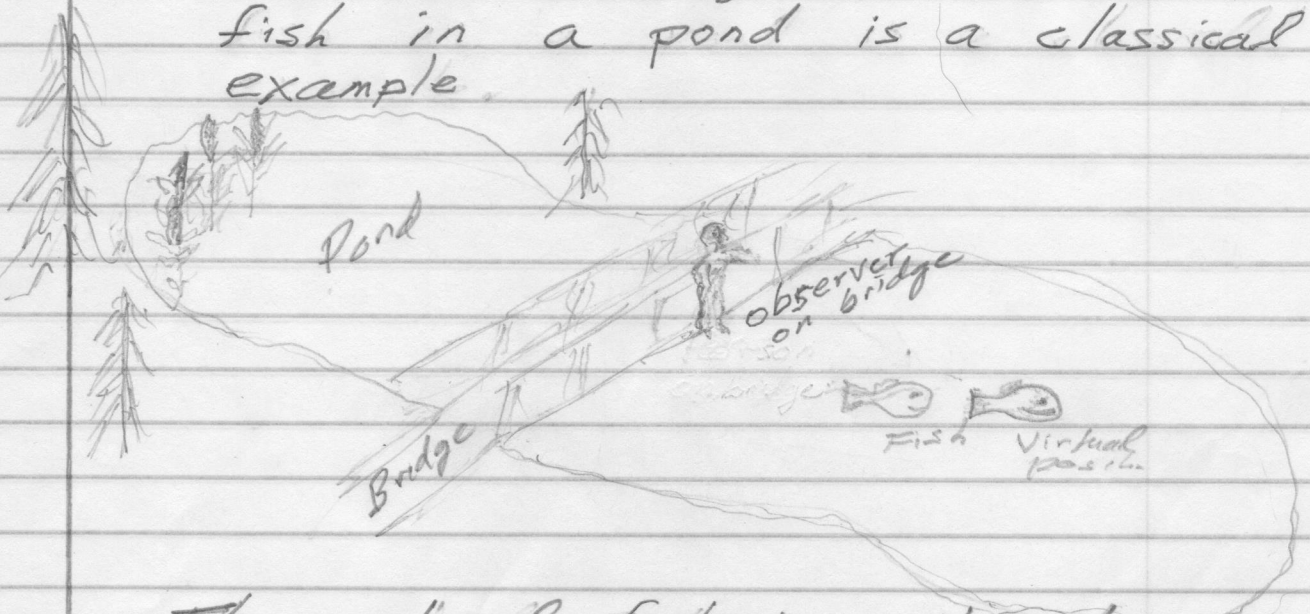
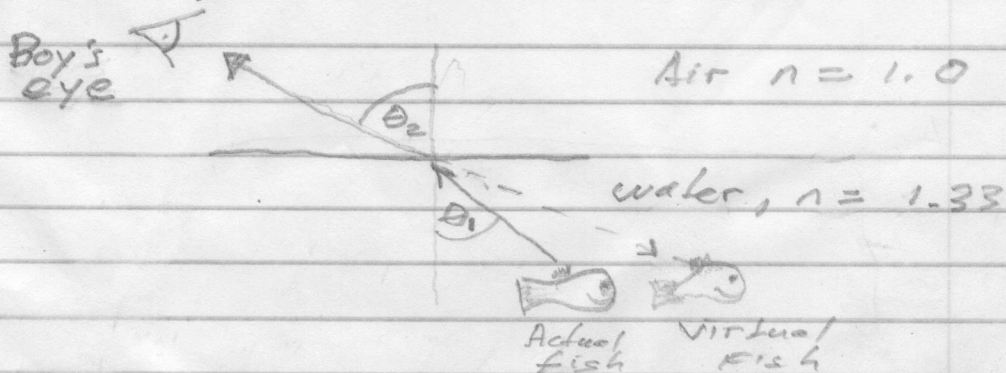


Ch. 1 Geometrical optics

In geometrical optics we make use of the fact that light bends when passing into a more dense (slower) medium. Looking down at a fish in a pond is a classical example.



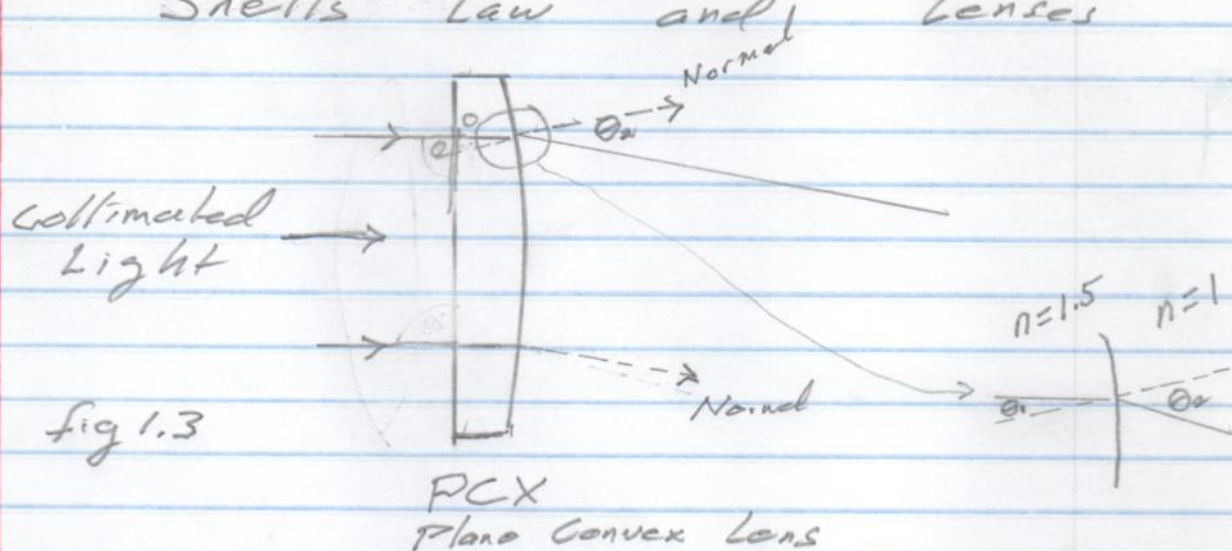
The actual fish is not where the observer sees the fish. Rather, the boy on the bridge is seeing a virtual image of the fish. This can make it tricky to cast a fishing lure to the exact fish location.



→ Snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$
 $\theta_2 = \arcsin \left(\frac{n_1 \sin \theta_1}{n_2} \right)$

Ch. 1 Geometrical Optics .2

Snell's Law and Lenses



$$\theta_i = 0^\circ, \text{ angle relative to normal!}$$
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

n_1 is now 1.5 glass

n_2 is now 1.0 air

$$\Rightarrow \theta_2 = \sin^{-1} \left(\sin \theta_1 \frac{n_1}{n_2} \right)$$

For $\theta_1 = 10^\circ$, see figure above

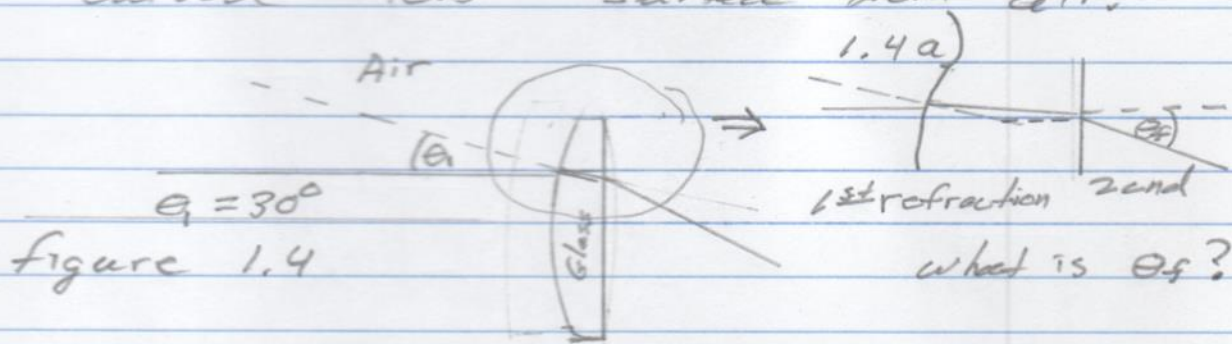
$$\theta_2 = \sin^{-1} \left((0.173) \cdot \frac{1.5}{1.0} \right) = \sin^{-1} \left(\frac{0.2595}{1} \right)$$

$$= 15 \text{ degrees}$$

So, for $n_1 > n_2$, θ_2 increases, refracts further from normal.

Ch. 1 Geometrical optics p.3

We examined $n_1 > n_2$, let's examine $n_1 < n_2$ or case of light incident on a curved lens surface from air.



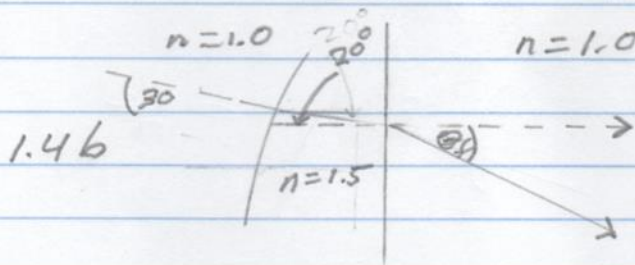
There are two refractions, the first is $n_1 < n_2$ so it's towards normal, for $\theta_i = 30^\circ$

$$n_2 \sin \theta_{r1} = n_1 \sin \theta_i$$

$$\theta_{r1} = \arcsin \left(\frac{(1.0) \cdot \sin(30^\circ)}{1.5} \right)$$

$$= 19^\circ$$

What, this is diverging? No, remember we have one more interface at the back of the lens!



$$n_2 \sin \theta_f = n_1 \sin \theta_{r1}$$

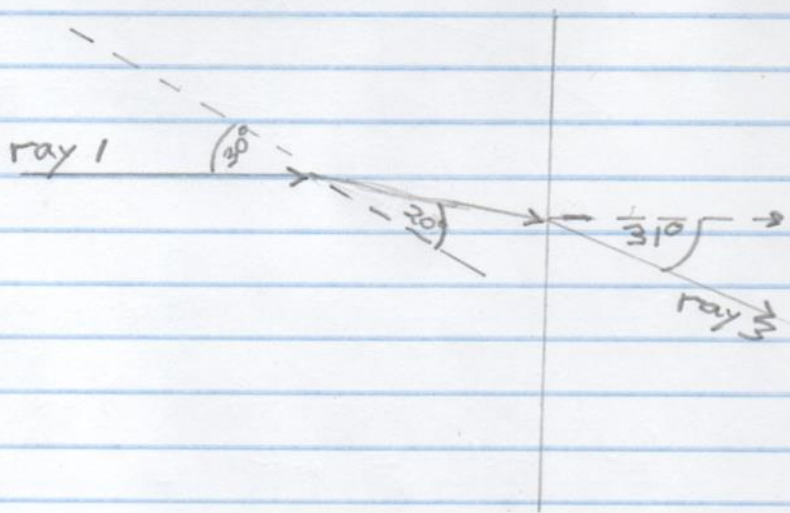
$$\theta_f = \arcsin \left(\frac{1.5 \sin 20^\circ}{1.0} \right)$$

$$= 31^\circ$$

Ch 1 Geometrical Optics p. 4

Two interface refraction continued.

This can't be, not much of a lens? Only 1° of refraction?
Let us draw the whole picture

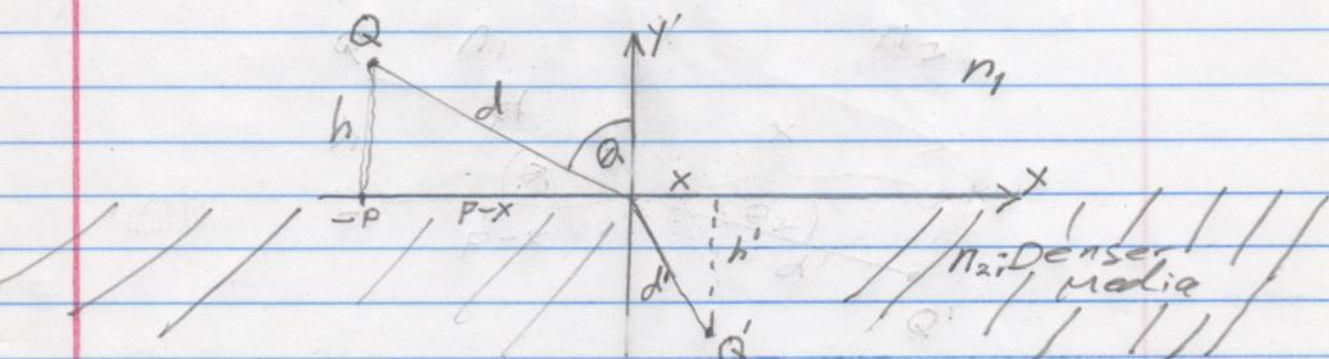


ray 1 and ray 3 have a 31° angle. So, there is well more than a net 1° change. That is because we need to examine the net effect of both refractors.

Ch. 1 Geometrical optics p. 5

Snell's Law continued.

A very nice physical derivation of Snell's law is performed using Fermat's principle of least time¹ for optical path. optical path = index · length



Path from $Q \rightarrow Q'$, denoted $[d] = n_1 d + n_2 d'$

$$d^2 = h^2 + (p-x)^2 \quad d'^2 = h'^2 + x^2$$

$$\text{"path", } [d] = n_1 \{h^2 + (p-x)^2\}^{1/2} + n_2 \{h'^2 + x^2\}^{1/2}$$

differentiate the path, let x vary and solve for a min or max

$$\frac{d}{dx} [d] = n_1 \frac{1}{2} u^{-1/2} \cdot \frac{du}{dx} + \frac{n_2}{2} v^{-1/2} \frac{dv}{dx}$$

$$u = h^2 + (p-x)^2 \quad v = h'^2 + x^2$$

$$\frac{du}{dx} = -2(p-x) \quad \frac{dv}{dx} = 2x$$

$$\frac{d}{dx} [d] = \frac{-n_1(p-x)}{\{h^2 + (p-x)^2\}^{1/2}} + \frac{n_2 x}{\{h'^2 + x^2\}^{1/2}} = 0$$

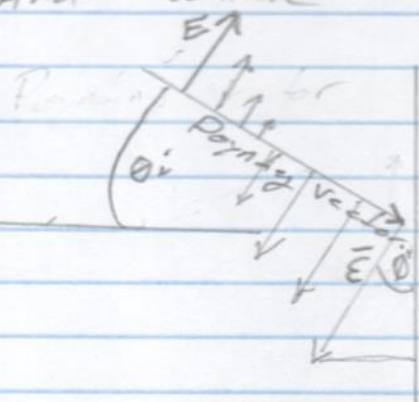
$$\Rightarrow \frac{n_1(p-x)}{195 \cdot d} = \frac{n_2 x}{195 \cdot d'} \Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2$$

¹ "Fundamentals of Optics," Jenkins & White, McGraw Hill, 1957 pp. 9-10

Ch. 1 Geometrical Optics p. 5

Snell's law, continued,

Physics students will recognize that Snell's law is merely the Maxwell Equation boundary condition that the \perp Displacement field is continuous. And where $n = \sqrt{\epsilon\mu}$



$$D = \sqrt{\epsilon\mu} \vec{E}$$

$$E_{\perp} = E \sin\theta_i$$

$$D_{\perp 1} = \sqrt{\epsilon_1 \mu_1} E \sin\theta_1$$

$$D_{\perp 2} = \sqrt{\epsilon_2 \mu_2} E \sin\theta_2$$

$$\Rightarrow \sqrt{\epsilon_1 \mu_1} E \sin\theta_1 = \sqrt{\epsilon_2 \mu_2} E \sin\theta_2$$

$$\text{since } \sqrt{\epsilon_1 \mu_1} = n_1, \sqrt{\epsilon_2 \mu_2} = n_2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Geometrical optics p.6

Ray Tracing

The snell's law approach is very precise, but what if we just want a picture of what is going on in an optical system? A "rough" idea. For this we can use the lens formula and ray tracing.

Lens formula for spherical surface: $\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$ Lens eq.

Derived in Jenkins + White "Fundamentals of optics" see 3.10 using snell's law and similar triangles.

And then applying this equation to a thin lens with two refractions, some of the refractive terms cancel out and we are left with:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{Thin lens equation}$$

s = object distance, s' = image distance

More commonly,

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

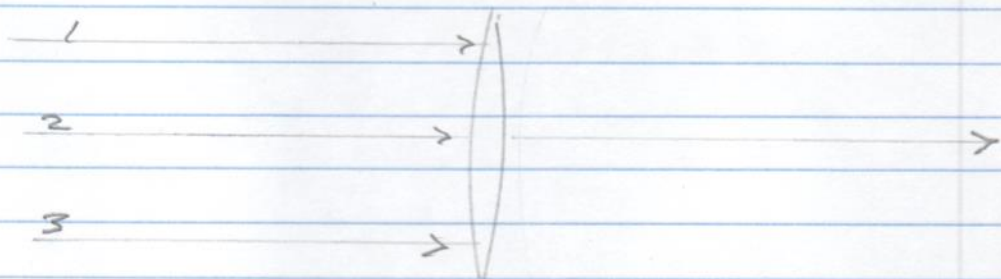
Note $1/f$ = curvature with radius f , so long focal length lenses have small curvature.

Geometrical optics p. 7

Ray Tracing continued,

We combine the thin lens equation with ray tracing to get a powerful technique in geometrical optics.

- a) Draw incident parallel rays, parallel to the optical axis

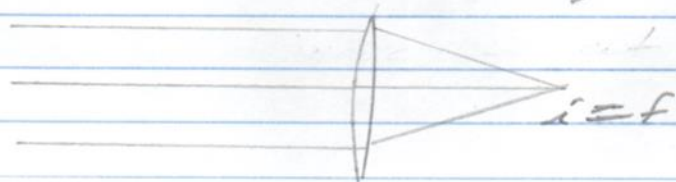


- b) Ray 2 is not refracted, from Snell's law $\theta_1 = 0^\circ \Rightarrow \theta_2 = 0^\circ$, the only answer as $\sin 0^\circ = 0$

- c) what is $\frac{1}{o}$ for parallel rays?

Recall that $\frac{1}{r} = \text{curvature of radius } r$. And for parallel rays the radius is infinite $\Rightarrow \frac{1}{\infty} = \text{curvature} = 0$.

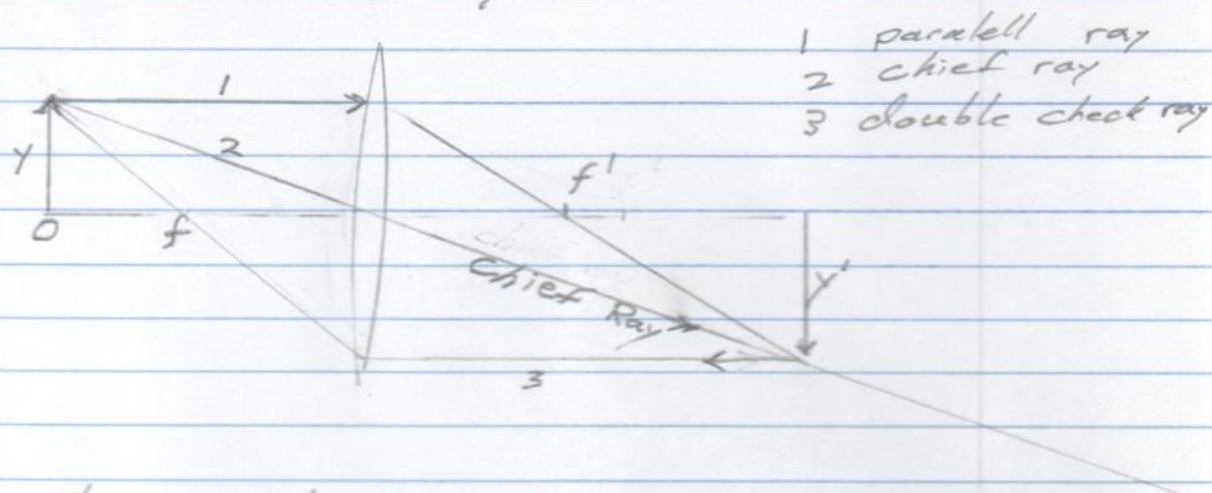
$$\text{Thus } \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{i} = \frac{1}{f} \Rightarrow i = f$$



Geometrical optics p. 8

Ray Tracing continued

We saw that rays parallel to the optic axis end up at the focal point. Now, let's talk about one more needed ray, it will be so important that we call it the chief.
The chief ray:



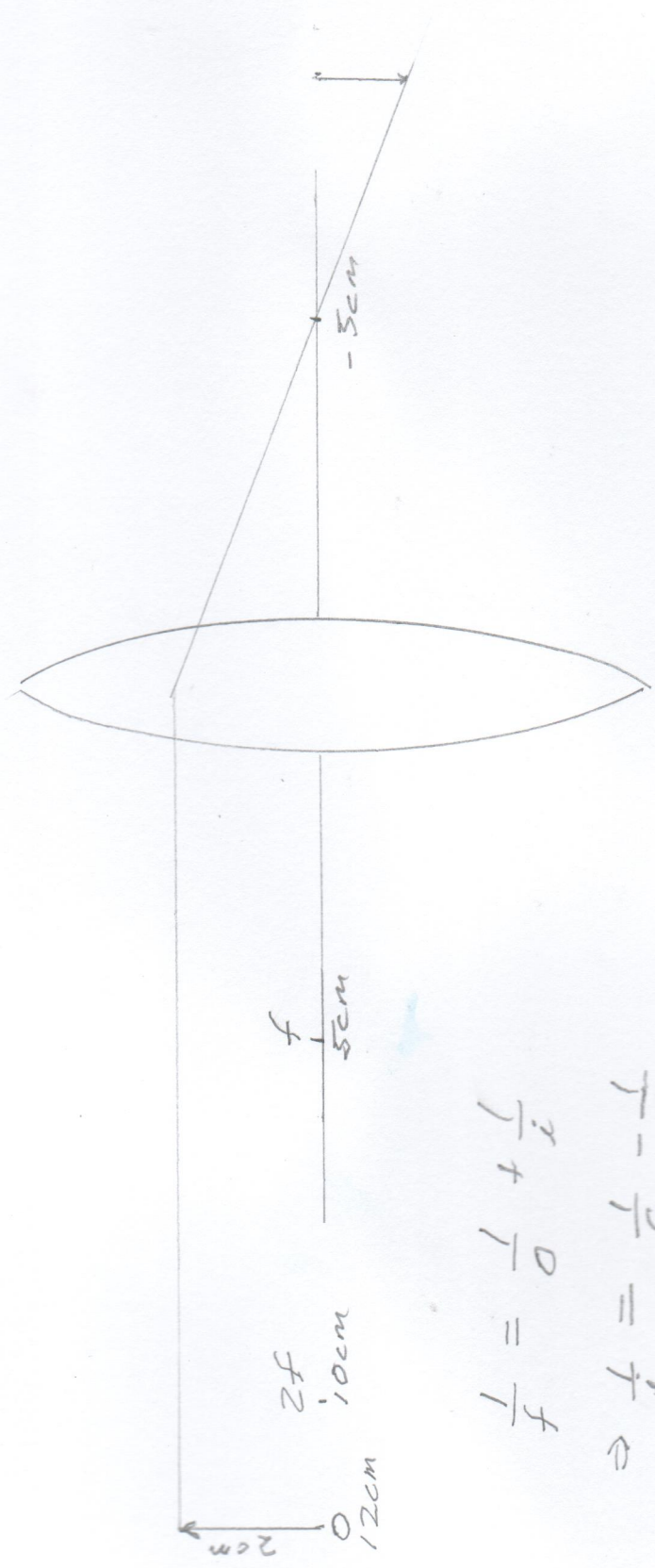
The chief ray goes through the center of the lens and because the normal is the same left interface and right interface, the chief ray is not refracted!

The chief ray and the parallel ray 1, will intersect at the image point. We check this with a 3rd ray that should go backwards from the image parallel to optic axis, through f and end on the object as it does!

Notice $y'/y > 1$, magnification. Now let's examine four cases with actual numbers -

Example I
 $o > 2f$
 minification

$f = 5\text{cm}$
 $o > 2f,$
 $o = 12\text{cm}$



$f > 0$, for convex lens

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$\Rightarrow \frac{1}{5} = \frac{1}{12} + \frac{1}{i}$$

$$\frac{1}{i} = \frac{1}{5} - \frac{1}{12}$$

$$= \frac{12 - 5}{60} = \frac{7}{60}$$

$$i = 60/7$$

$$i = 8.6\text{cm}$$

Object is minified?
 When is this useful?

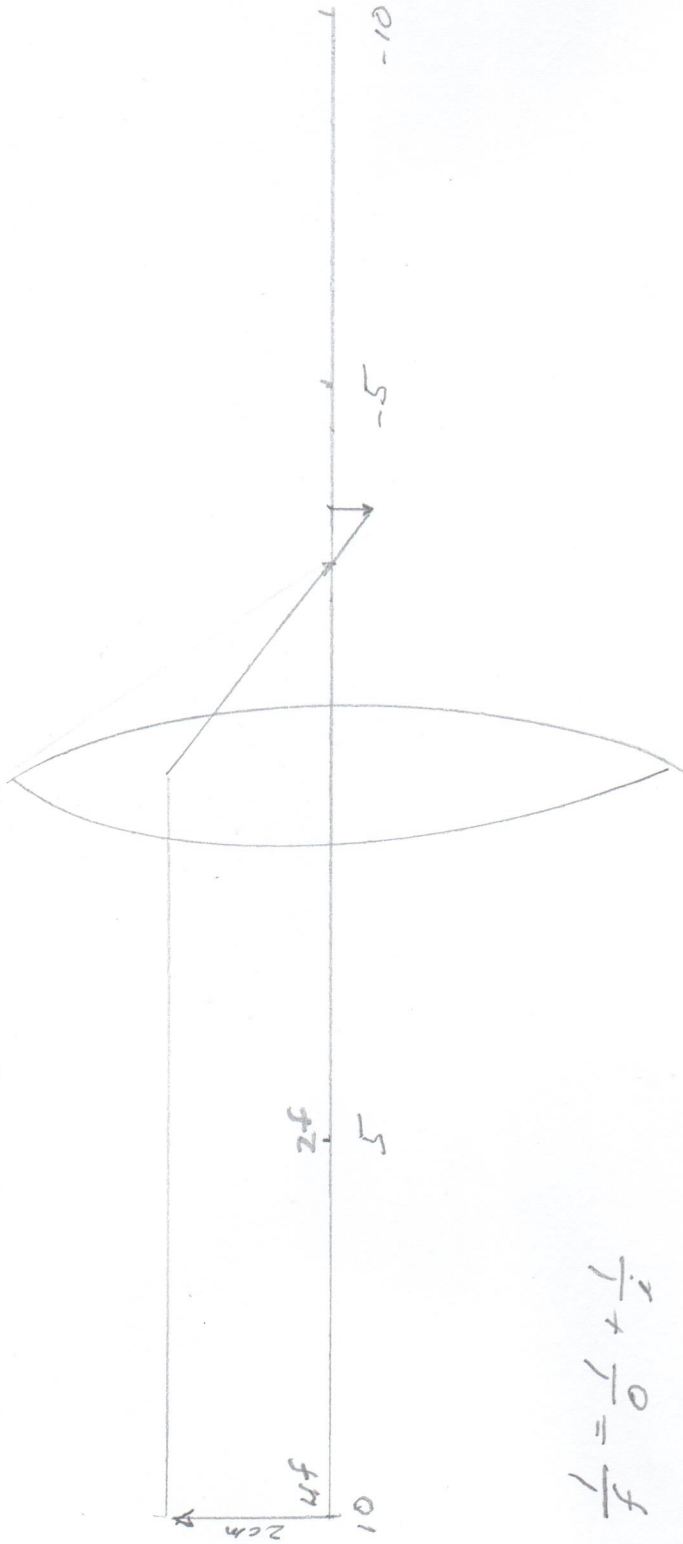
$1/5 = 1/12 + 1/i \Rightarrow i = 18.53\text{cm}$

example $\frac{1}{0} = \frac{1}{f}$

$$f = 2.5 \text{ cm}$$

$$0 > 2f$$

$$0 = 10 \text{ cm}$$



$$\frac{1}{f} = \frac{1}{0} + \frac{1}{i}$$

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{0}$$

$$\frac{1}{i} = \frac{1}{2.5} - \frac{1}{10}$$

$$\frac{1}{i} = \frac{4}{10} - \frac{1}{10} = \frac{3}{10}$$

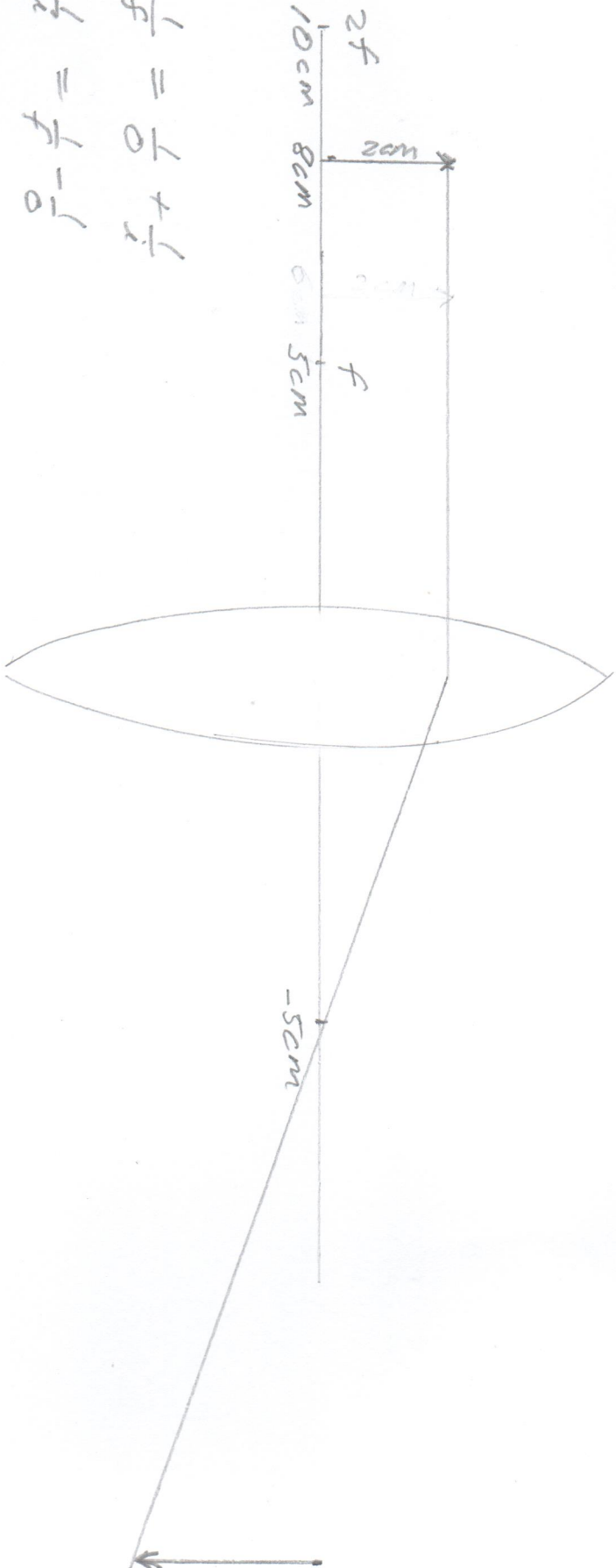
$$\Rightarrow i = 10/3 = 3.33 \dots$$

object is further minimized.

Example III

$f < 0 < 2f$
Magnification
Real
inversion

$$f = 5 \text{ cm}$$



$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$\frac{1}{5} = \frac{1}{8} + \frac{1}{i}$$

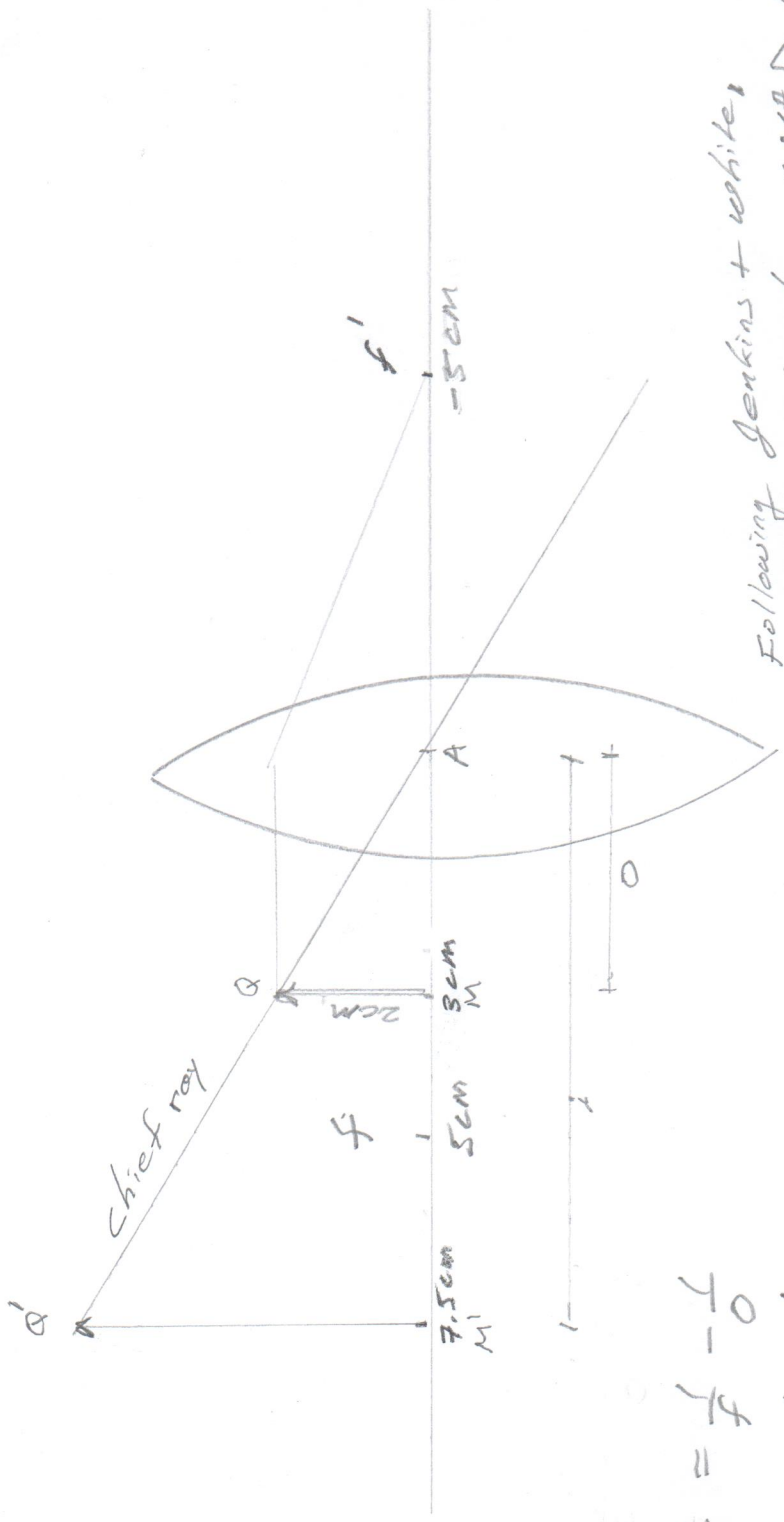
$$= \frac{1}{5} - \frac{1}{8}$$

$$= \frac{8 - 5}{40} = \frac{3}{40}$$

$$= \frac{3}{40}$$

$$i = 13.33 \text{ cm}$$

Example IV
 $0 < f$
 Magnification
 and
 Not inverted



$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{5} - \frac{1}{3}$$

$$= \frac{3}{15} - \frac{5}{15}$$

$$= -\frac{2}{15}$$

$$v = -7.5 \text{ cm}$$

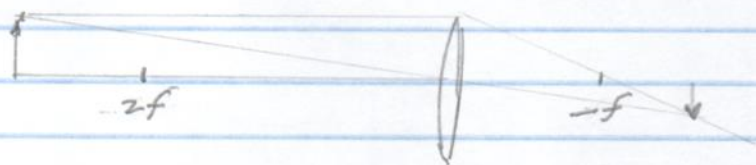
Following Jenkins + white,
 similar triangles $\triangle MA \triangle / \triangle M'A'D$
 $\Rightarrow \frac{Q'M'}{QM} = \frac{AM'}{AM}$
 But $\frac{Q'M'}{QM}$ is lateral magnification, M
 $\frac{Q'M'}{QM} = M = \frac{-v}{u} = \frac{7.5}{3} = 2.5$
 Magnification of 2.5

Ch. 1 Geometrical optics 13.

Now that we have done many examples minification, magnification, virtual images, and cases where the image is inverted $f < 0 < 2f$, let us talk about an electron microscope using all of these concepts.

First key to good high resolution imaging is to have the smallest possible source. The modern field emission gun (FEG) a very small source. But we can use our optics to make it even smaller and better able to resolve.

In example I page 9 we had the case of $0 > 2f$ and the resultant image was "minified." This is ideal for the condenser system



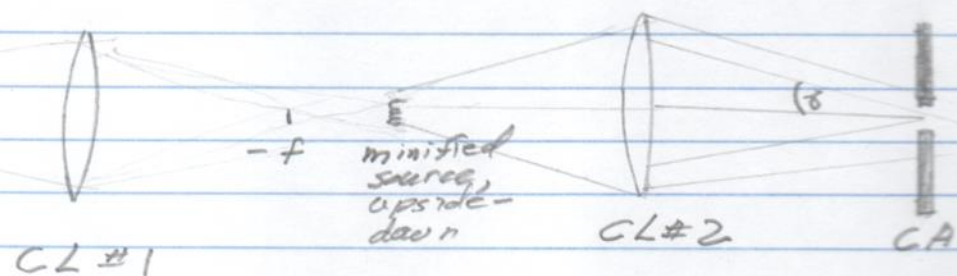
Condenser Lens 1, spot size on a microscope.

By making the lens stronger we can demagnify even more. As discussed in Fultz + Howe, the strength of C.L. 1 goes up the demagnification increases.

1 Fultz + Howe, Transmission Electron Microscopy and Diffracton Materials, Springer Verlag, p. 85

Ch. 1 Geometrical optics 14.

Now that we have a demagnified source via condenser lens 1, we put it in the focal plane of condenser lens 2. By varying the strength of the condenser lens 2, we can control the beam convergence on the specimen. We also place an aperture, condenser aperture in the beam to a) reduce current and b) clean up the beam for higher resolution imaging - particularly critical in STEM work.



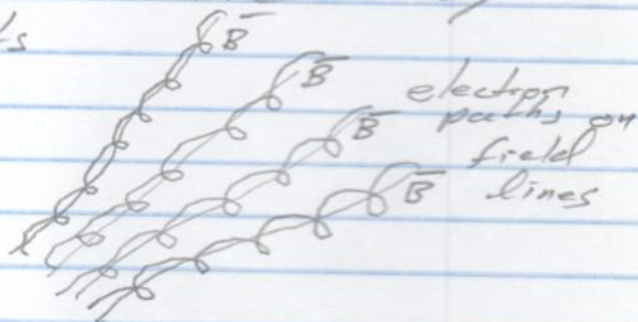
Condenser aperture eliminates some aberrations in the condenser system and controls the illumination angle, α . Alpha is one of the two very important angles in TEM. The other angle of importance will be β , we control that with the objective aperture and will talk about that in a bit. Smaller C_2 also increases the contrast as the illumination is more parallel
Z Williams & Carter p. 134

Geometrical optics page 15

Aside, we have been analyzing the lenses in a TSM with a ray tracing approach. We need to realize that electromagnetic lenses work a little differently. They work on the Lorentz force that a charged particle feels in an electromagnetic, B , field:

$$\text{Lorentz Force} = q \left(\vec{v} \times \vec{B} \right)$$

In fact, these electrons follow helical trajectories and these trajectories are deflected by field gradients



We can solve for the radius of these gyrations by equating the centripetal acceleration to the Lorentz force.

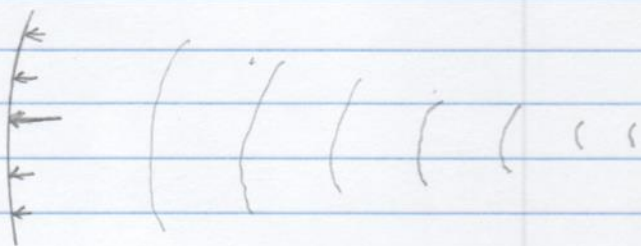
$$\frac{qvB}{c} = \frac{mv^2}{R}$$

$$R_L = \frac{cmv^2}{qvB} = \frac{mcv}{qB}$$

R_L = Larmor radius, and is inverse with Magnetic field. Tighter trajectories correspond to stronger fields.

Geometrical optics ch. 1 p. 16

Now that we realize the electrons don't really travel as rays but as Helices, we can visualize an electromagnetic lens in the TEM as a phase shifter.²

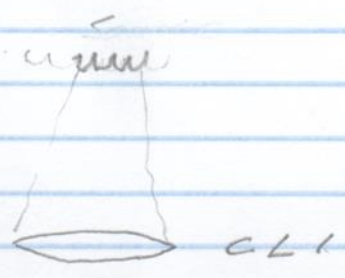


Lens as a phase shifter. Central portions of the lens retard the phase more than near top and bottom. This causes the waves to focus at the right.

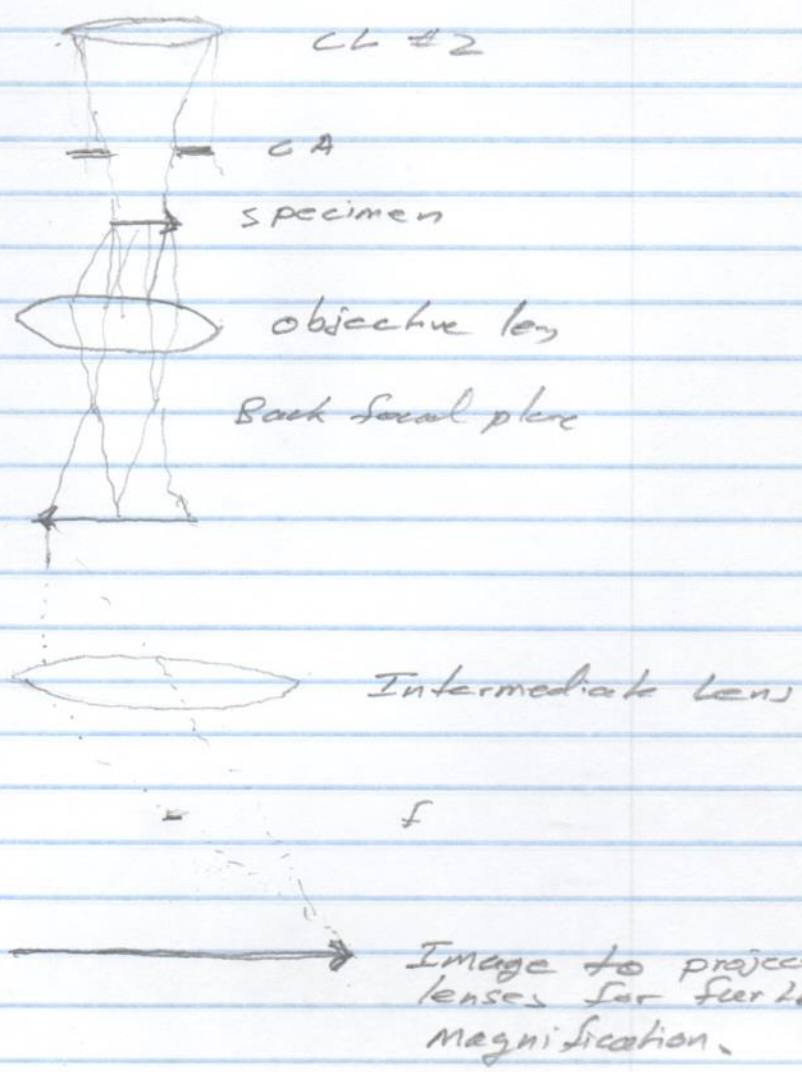
But, for all the lack of physics in the ray tracing method, it is still a useful tool for visualizing paths of electrons in a TEM. Let us now put all of the lenses together to form a diagram of a simpler, 2 condenser, TEM. We do that on the next page.

2. Fultz and Howe, Transmission Electron Microscopy and Diffractometry of Materials Springer p. 92
4th ed. 2013

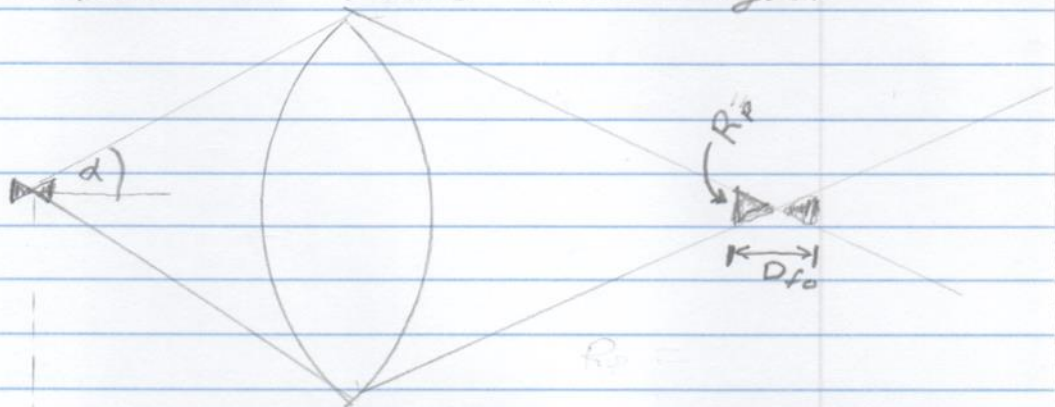
TEM Ray Tracing



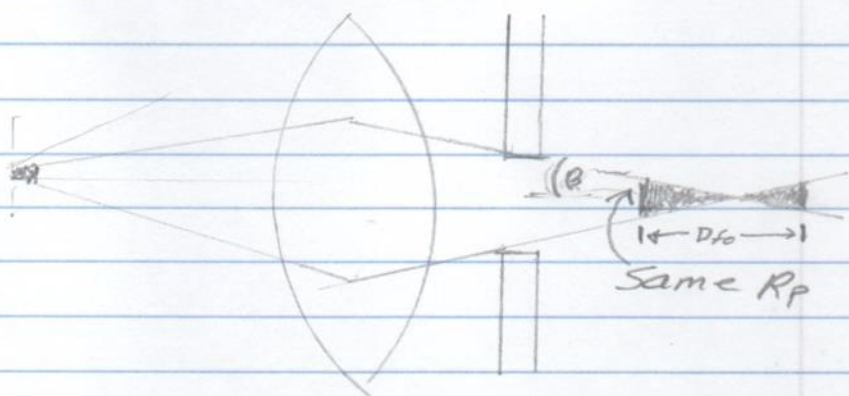
minified source



The effect of objective aperture on the depth of field. From Flegler³



With objective aperture:



By limiting the angle collected, β , we increase depth of field.

$$\text{Depth field} = d/\alpha, \text{ with } \alpha = \beta$$

For $d = .1 \text{ nm}$ (Titan TEM at OSC)
 $\alpha = 10^{-3} \text{ rad}$

$$\text{Depth field} = .1/10^{-3} = 100 \text{ nm}$$

So, the entire thickness of TEM sample should be in focus.

3. Flegler, "scanning and transmission Electron microscopy an Introduction, Oxford p. 48