Chapter 2: Elements of Fourier
or Physical Optics

1) Motivation:

Carl Zeiss commissioned Abbé
for a better microscope. Abbé
failed as he used a
small aperture system. He
realized diffraction was a
nuisance and went to a larger
den. He succeeded.

\[
\text{Intensity is given by:}
\]

\[
\text{Period } = \frac{1}{2\sin \theta}
\]

\[
\tan \theta = \frac{D}{2f} - \frac{D}{2F}
\]

Small angle: \( \theta \approx \frac{D}{2F} \)

\[
\sin \theta = \frac{D}{2F}
\]

Period or Spatial Frequency:

\[
\frac{\lambda}{D} = \frac{A}{f}
\]

Similar to previous results.
Fourier optics, light field is written as a superposition of plane waves propagating in different directions. A distribution of angles is akin to angular frequency.

\[ \angle \beta \]
\[ \angle \alpha \]
\[ \angle \gamma \]

The wavelets, Huygens principle, are comprised of a superposition of angles. The result of len depends upon the spatial frequency.

\[ u = \frac{x}{f\lambda}, \quad v = \frac{x}{f\lambda} \]

Since \( f = \frac{1}{\text{[length]}} \) and \( \lambda = \frac{1}{\text{[length]}} \)

\[ u = \frac{1}{\text{[length]}} \]

Elements of Fourier optics or Physical optics p.2
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Fresnel Diffraction integral:

\[ S_f = \left( \frac{E_0 c^{i\kappa^2/2}}{i\alpha f} \right) \int_{[x(y), y]} e^{i\alpha(x,y)} dx dy \]

We can split Fresnel integral into two parts:

\( f_1(x,y) = \) radial incident on lens
\( f_2(x,y) = \) transmission of lens.

\[
\text{with convolution theorem:} \\
\int S_\lambda e^{i\alpha(x,y)} = \int \frac{E_0 c^{i\kappa^2/2}}{i\alpha f} \int_{[x(y), y]} e^{i\alpha(x,y)} dx dy.
\]

\[
= \int \frac{E_0 c^{i\kappa^2/2}}{i\alpha f} \int_{[x(y), y]} e^{i\alpha(x,y)} \cdot \text{P.S.} dx dy.
\]

For a circular lens:

Point spread, \( p(x,u) = \frac{\pi i \alpha^2}{\alpha^2} \text{jinc} \left( \frac{\pi D^2}{\alpha^2} \right) \)

\( \text{jinc} \) is Bessel function analog to \( \text{sinc} = \frac{\sin \pi x}{\pi x} \)

1. Adams & Hughes "Optics 2nd Ed." from Fresnel to Fourier, Oxford, 2019
Elements of Fourier Optics

For a plane wave on a lens, the solution is simple:

\[ F(x, y') = e^{ix} \]

\[ F(e^{i\phi}x) = \delta(x) \] Delta function

For two plane waves, are separated by \( \Delta \theta \):

\[ F \left[ e^{i2\pi \Delta \theta (x')} \right] (x) = \delta \left( u - \frac{\Delta \theta}{\lambda} \right) \]

where \( 2\pi (u - \Delta \theta) = \frac{\lambda}{\lambda} \) \( u = \frac{x}{\lambda f} \)

Plugging this into Fresnel eq.

\[ E_f = \frac{E_0 e^{i\phi r}}{1 + \frac{1}{2} \frac{\Delta \theta}{\lambda} \cdot P.S.F.} \]

For a lens of diameter \( D \) and focal length \( f \):

\[ P.S.F. = j \text{inc} \left( \frac{\pi D \phi}{\lambda f} \right) \]

Intensity \( E \times E^* \)

\[ = \frac{E_0 \pi^2 D^2}{16 f^2} j \text{inc}^2 \left( \frac{\pi D (u - \Delta \theta)^2 + v^2}{\lambda} \right) \]

Air Pattern code displaced to \( x = \Delta \theta f \)
see eq 9.5 Adams + Hughes
So, the two plane waves of slightly different incident angle, $\theta_1$, give us a nice result. The observed intensity in the focal plane is two Air patterns. What is the resolution of said system?

Rayleigh criteria:

Airy $\pm 1$

![Airy pattern diagram]

Air $\pm 2$

![Air pattern diagram]

$$f \Delta \theta = \frac{1.22 \lambda}{D} \quad \text{(left as homework)}$$

and $$\Delta \theta_R = \frac{1.22 \lambda}{D}$$

So, we get smaller $\Delta \theta$, better resolution when $D$ is increased. That is, when more spatial frequencies are submerged by the lens.
Spatial filtering.

We learned that resolution increases when we increase the diameter of the lens; this confused Abbé early on.

Can we alter the resolution of a system by changing the spatial frequencies that get to the lens? Yes, concept of apodization, to cut feet off a pod = without feet.

We do this in optical holography, "clean up" the laser beam with a strong lens 1, objective eyepiece, and a pin hole aperture at focal point.
Fourier optics

We see examples of the Fourier transform of the lens in appendix II, of physical optics.

Appendix II. Fig 1 sinc (spatial)
rectangular aperture

Fig 2 Bessel functions, note different orders and symmetry.

Fig 3 Jinc function, $\mathcal{F}^{-1}$ of round lens.

From the result of Fig 4, we see the Jinc and sinc function are very similar. So if you are familiar with $\sin(x) = \frac{\sin(\pi x)}{x}$ please use that analogy.

Airy function, the intensity of the field squared at field pt.

$E^2 = E_0^2 \frac{i k h}{4\pi} \int [f(x',y')] (u,v) \mathcal{F}^* [psf(u,v)] \, du \, dv$

for plane wave $f(x,y) = E_0 \exp(i k x + i \lambda y)$

$\psi^2 (0,0) = \frac{1}{\pi k} \sin^2 \left( \frac{\pi d}{\lambda} \right)$

where $E_0 \equiv \frac{1}{\sqrt{\pi k d}}$
$\frac{\sin(x)}{x}$ or the Sinc function
J1 Bessel function over x or "Jinc" function
Sinc function in red and Jinc function in cyan

In optics $x = \pi D \rho / \lambda f$. $D =$ lens diameter, $\rho = \sqrt{x^2 + y^2}$, $f =$ focal length, $\lambda =$ wavelength.
Fourier optics p. 8

But if transform of a plane wave = $S$ function

$i. \quad E_f = \frac{E_0 e^{i \frac{2\pi}{\lambda} s(u-\Delta s) \cdot \text{psf}(u)}}{i \sigma f}$

For a plane wave inclined at an angle $\theta$ to normal incidence. This example used in Adams & Hughes as an example of angular resolution.

$\Rightarrow I(f) = \frac{I_0 \pi^2 D^4}{16 \lambda^2 z^2} \sin^2 \pi d \frac{(u-\Delta s)^2}{\lambda^2}$

See appendix II fig 4.

Ry. pattern + fig 5 have overlapping diff. patterns.

The angular resolution of a lens is the minimum separation of the Air patterns or when $\frac{1}{2}$ of one pattern is upon min of other.

$\Delta \theta_{\text{resolution}} = \frac{1.22 \lambda}{D}$

This is what Abece discovered. We need a bigger lens to improve the microscope.
Airy disc or square of Jinc in cyan and Jinc displaced red

\[ x = \pi D \rho / \lambda f \]

X: 3.812
Y: 4.399e-06
Inverse Averaging: Super Resolution

You can actually block the central part of a diffraction pattern (image) of lens 1 with an annular (french) filter to get an image that with lens 2, to form an image with only the higher spatial frequencies and increase the resolution.

Conversely, in dark field TEM we select only a few spatial frequencies. This eliminates the higher spatial frequencies and so the image has lower resolution.

[Diagram of microscope setup with labels: sample, objective lens, back focal plane, objective aperture, image plane, camera.]
Fourier optics

Relationships between Fourier optics concepts and terminology and electron microscopy.

Does the point spread function of page 4, \( F(x, y) \), which is typically \( \frac{1}{2} D^2 \cdot jinc \left( \frac{PD}{AF} \right) \), Adams etc.

Equate to contrast transfer function in electron microscopy of spence or content Williams.

Yes, spence has similar approach but the psf, point spread function of Fourier optics is modified by a term that represents the strong influence of spherical aberration in electron optics.

Transfer function \( A(u, v) = R(u, v) \exp(i2\pi u) \) in spence 3.24.9

Pupil
Aberration
Transfer
Spherical
High spatial
Holography
Williams and Carter, intensity gcxy:

\[
g = I(x, y) = \sum G(u_x, v_y) \exp 2\pi i(xu_x + yv_y)
\]

Sine over spatial frequency or for FTTEM we are reciprocal lattice vector.

G.T.F. \[ G(u) = \frac{H(u) F(u)}{4} \text{ or } \frac{A(u) B(u)}{4} \text{ or } \frac{C(u) D(u)}{4} \]
Relevance of FT Fourier optics to modern electron optics terms:

In electron microscope, the electron beam (charged particle) interacts with the charge in the sample. This changes the phase of the wave electron waves. This is a more complicated interaction than the ray charged photon (light) passing through a sample at a lens.

The connection of Fourier optics to electron microscopy is particularly strong in the TEM. In the back focal plane of the objective lens we have the diffraction pattern or the angular information.

Beam is scattered by sample

Back focal plane

Scattered beams from diffraction collected in SPD

Image plane

Now, back to Fourier optics
Fourier optics

Relationship of Fourier optics to modern electron optics terms:

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The connection of Fourier optics to electron microscopy is particularly strong in the TEM. In the back focal plane of the objective lens we have the diffraction pattern or the angular information of the sample.

Now, back to Fourier optics.
Example, 4f spatial filter.

Or, cleaning up a Gaussian laser beam. On the bottom of page 4, I mentioned cleaning up a laser beam with a strong lens and pinhole apertures. Now we see the math of this.

Input intensity

\[ f(x', y') = e^{-\frac{(x'^2+y'^2)}{2A_0^2}} (1 + \epsilon \cos \left( \frac{2\pi x'}{d} \right)) \]

where \( A_0 = \) Gaussian beam width
\( d = \) spacing between the dirty little peaks above.

Fresnel integral:

\[ \psi(t) = \frac{E_0 e^{i k t}}{i k t} \text{FT} \left\{ \left[ f(x', y') \right] \mu \nu \right\} \]

9.1

\[ \text{FT} \left\{ f(x', y') \mu \nu \right\} = \text{FT} \left\{ e^{-\frac{(x'^2+y'^2)}{A_0^2}} (1 + \epsilon \cos \left( \frac{2\pi x'}{d} \right)) \right\} \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i (ux+vy) - \frac{(x'^2+y'^2)}{A_0^2}} \left\{ 1 + \epsilon \cos \frac{2\pi x'}{d} \right\} d\nu d\xi \]

This is hard to integrate, so let's examine the convolution approach.
Cleaning a gaussian beam continue:

Now carry out the convolution:

\[ E_f = \frac{E_0 e^{i k r}}{i k f} \left\{ \frac{\delta(u) + \delta(u - d/2) + \delta(u + d/2)}{\pi A_0^2 e^{-\pi (u^2 + v^2) A_0^2}} \right\} \]

\[ A_0 = \text{width of Gaussian beam} \]

take only central spatial freq

set \( \frac{A_0}{d} = 5 \) \( \Rightarrow \frac{1}{d} = 5A_0 \)

This eliminates the two spots
from the spatial functions

\[ E_f = \frac{E_0 e^{i \frac{2 \pi}{2 A_0^2} \frac{2 \pi}{2 A_0^2}}} {v(u)} \]

Now Fourier Transform again
at the first focal plane
Fourier optics

If spatial filter continued, Gaussian beam clean up:

After the second lens, one more F.T.:

\[ \phi(x, y) = \frac{1}{\sqrt{\pi}} \int \int e^{-i2\pi(x'x + y'y)} e^{-\frac{r^2}{2\sigma^2}} \, dx'dy' \]

\[ \phi(x, y) = \frac{1}{\sqrt{\pi}} \int \int e^{-i2\pi(x'x + y'y)} e^{-\frac{r^2}{2\sigma^2}} \, dx'dy' \]

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{r^2}{\sigma^2}} \, dx'dy' \]

Compared with 9.16 Adams Chapter 3.

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{r^2}{\sigma^2}} \, dx'dy' \]

Consider the integral over \( dx'dy' \)?

Sec 9.6 Adams Chapter 3.

Note Sec 9.6 Adams Chapter 3.

\[ u = \frac{x}{fa}, \quad v = \frac{y}{fa} \]

Now, use \( e^{au^2} = \cos a - i \sin a \)

Integral over sine variables:

Then Abramowitz Stegun 10.1.2 p. 302

\[ \int e^{-ax^2} \, dx = \sqrt{\frac{\pi}{a}} e^{-\frac{x^2}{4a}} \]

\[ \int e^{-\pi u^2} \, du = \sqrt{\frac{\pi}{\pi a^2}} e^{-\frac{u^2}{2a^2}} \]

\[ \int e^{-\pi u^2} \, du = \sqrt{\frac{\pi}{\pi a^2}} e^{-\frac{u^2}{2a^2}} \]

\[ \int_{-\infty}^{\infty} e^{-\pi x^2/a^2} \, dx = \sqrt{\pi} e^{-\pi^2 x^2/a^2} \]

\[ \int_{-\infty}^{\infty} e^{-\pi u^2} \, du = \sqrt{\frac{\pi}{\pi a^2}} e^{-\frac{u^2}{2a^2}} \]

\[ \int_{-\infty}^{\infty} e^{-\pi u^2} \, du = \sqrt{\frac{\pi}{\pi a^2}} e^{-\frac{u^2}{2a^2}} \]

\[ A_0 = \text{width of Gaussian, so } x^2/A_0^2 \text{ has no dimensions.} \]

Thus, \( E = (\frac{a}{2i}) \int_{-\infty}^{\infty} e^{i2\pi(x'x + y'y)} e^{-\frac{r^2}{2\sigma^2}} \, dx'dy' \)

\[ E = (\frac{a}{2i}) \int_{-\infty}^{\infty} e^{i2\pi(x'x + y'y)} e^{-\frac{r^2}{2\sigma^2}} \, dx'dy' \]
Gaussian clean up with spatial filter continued.

\[ \psi = \text{constant} \times e^{-\frac{4 \pi^2 y^2}{W^2}} \]

where \( W_0 \) was original gaussian envelope

Picture of what we did:

- Perfect Gaussian beam, Ideal for holography or optical computing.
Fourier optics  P. 16

\[
\mathcal{F}\{e^{i(ux+vy)}\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(ux+vy)} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \, dx \, dy
\]

but \( \mathcal{FT}\{f(x,y)\}(u,v) = \mathcal{FT}\{\text{Gauss}(x,y)\} \cdot \text{mod}(x) \)

where \( \text{Gauss}(x,y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}} \)

\( \text{mod}(x) = 1 + 6 \cos \frac{2\pi x}{a} \)

Use inverse convolution theorem:

\[
\mathcal{FT}\left[ g(x) * f(x) \right] = \text{convolution of } \mathcal{FT}(g) \cdot \mathcal{FT}(f)
\]

convolution, \( h(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) * f(x-x') \, dx' \)

so \( g(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(ux+vy)} A_0^2 \, dx \, dy \)

\( \gamma = (\sqrt{\pi}) A_0^2 \)

and \( f(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i (ux+vy)} (1 + 6 \cos \frac{2\pi x}{a}) \, dx \, dy \)

\[ f(u,v) = \delta(v) \left\{ \delta(u + \frac{1}{2} \left( \frac{8\pi v^2}{a^2} + \frac{8\pi u}{a} \right) + \delta(u - \frac{1}{2} \left( \frac{8\pi v^2}{a^2} - \frac{8\pi u}{a} \right) \right) \right\} \]
Fourier optics

Now:

$$\mathcal{F} \left[ g(x) f(x) \right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x') f(x-x') dx'$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\pi} a^2 e^{-\pi^2 a^2 (u'^2 + v'^2)} \, du' dv'$$

$$\delta(v-v') \left\{ \delta(u-u') + \frac{1}{2} \left( \delta(u+v-u') + \delta(u-v-u') \right) \right\}$$

Our spatial filter limits the frequency in the convolution.

$$\Delta_0 = 5d$$

$$\int_{-\infty}^{\infty} dv' \int_{-\infty}^{\infty} I(u') du' = \int_{-\infty}^{\infty} dv' \int_{-\infty}^{\infty} I(u') du'$$

Note \( u = \frac{x}{\lambda}, \quad v = \frac{y}{\lambda} \)

The sifting property of the delta function selects only the central diffracted spot.

$$\frac{\partial^2}{\partial T} = -\frac{\lambda^2}{\pi} A_0^2 e^{-\pi^2 A_0^2 (u'^2 + v'^2)}$$

This is the input to the last lens. Next, we solve for E-field after final lens.
$F \propto \mathcal{S} f = \frac{E_0 e^{i\pi r}}{i\lambda f} \mathcal{F} \left[ \sqrt{\frac{\pi}{\lambda^2}} A_0^2 e^{-\pi^2 \lambda^2 x^2} \right]$

Again $\mathcal{F} \text{Gaussian} = \text{Gaussian}$

$\therefore \hat{f} = \frac{A_0^2 \sqrt{\pi}}{i \lambda f} \int_{-\infty}^{\infty} e^{-\pi^2 \lambda^2 x^2} e^{i \pi u x} e^{-\pi^2 \lambda^2 y^2} e^{i \pi v y} dx$

$u = \frac{x}{\lambda f} \quad v = \frac{y}{\lambda f}$

$dv = \frac{1}{\lambda f} dy \quad du = \frac{1}{\lambda^2 f^2} dx$

$dy = \frac{2 \lambda f}{\lambda^2 f} du \quad dx = \lambda f du$

$\therefore \hat{f} = \frac{E_0 A_0^2 \sqrt{\pi}}{i \lambda f} \int_{-\infty}^{\infty} e^{i \pi u x} e^{-\pi^2 \lambda^2 x^2} e^{i \pi v y} e^{-\pi^2 \lambda^2 y^2} e^{-2 \pi \lambda f x} e^{-2 \pi \lambda f y} dx$

$\mathcal{F} \text{Gaussian} = \mathcal{F} \left[ e^{-\pi^2 \lambda^2 x^2} \right] = \sqrt{\frac{\pi}{\lambda^2}} w_0 e^{-\pi^2 \lambda^2 x^2}$

$\frac{1}{2} + \frac{1}{w_0^2} = \lambda^2 A_0^2$

$\therefore \hat{f} = \frac{E_0 A_0^2 \sqrt{\pi}}{i \lambda f} \mathcal{F} \left[ e^{-\pi^2 \lambda^2 x^2} e^{-\pi^2 \lambda^2 y^2} \right] \int_{-\infty}^{\infty} e^{-\pi^2 \lambda^2 x^2} e^{-\pi^2 \lambda^2 y^2} e^{-2 \pi \lambda f x} e^{-2 \pi \lambda f y} dx$
\[ S_f = -i \frac{\varepsilon_0}{\sqrt{2\pi}} \frac{A_0^2}{V_{w_0}} \sqrt{\frac{x^2}{2\pi w_0^2 x^2} - \frac{\pi^2 w_0^2 y^2}{2\pi}} e^{i2\pi x^2/\lambda} e^{i2\pi y^2/\lambda} \]

\[ \Rightarrow A_0^2 = \frac{1}{\pi w_0^2} \]

\[ S_f = -i \frac{\varepsilon_0 A_0^2 \pi w_0^2}{\pi} e^{i2\pi x^2/\lambda} e^{i2\pi y^2/\lambda} e^{-(x^2+y^2)/A_0^2} \]

\[ \left( \frac{x^2+y^2}{2} \right) / A_0^2 \]

Now, \[ A_0^2 \pi w_0^2 = \frac{\lambda_0}{\lambda} \frac{1}{\pi^2 A_0^2} = \frac{1}{\lambda} \]

\[ S_f = -i \frac{\varepsilon_0}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2 \pi}} e^{i2\pi x^2/\lambda} e^{i2\pi y^2/\lambda} e^{-(x^2+y^2)/A_0^2} \]

And so, intensity \[ S = S_f * S_f \]

\[ \frac{\varepsilon_0}{\sqrt{2\pi}} e^{-i2\pi x^2/\lambda} e^{-(x^2+y^2)/A_0^2} \]

\[ \left( \frac{x^2+y^2}{2} \right) / A_0^2 \]

\[ = \frac{\varepsilon_0^2}{2\pi} e^{-2(x^2+y^2)/A_0^2} \]

Result is a clean gaussian beam.
Aside, some useful Fourier Transform bits:

1) May need use of a trig identity
   a) $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$
   b) $e^{-i\phi} = \cos \phi - i \sin \phi$

   $2 \cos \theta = 1 + \cos 2\theta$
   $\cos 2\theta = 2\cos^2 \theta - 1$

2) Spatial frequency
   $u = \frac{x}{\lambda}$
   $s \nu = \frac{x}{\lambda}$

3) $FT[\delta(x) u] = \delta(u)$
   $FT$ denotes Fourier Transform

Convolutions:
   $FT(f * g)(x) = FT[f(x)g(x)]$

$= \int_{-\infty}^{\infty} f(x)g(-x)e^{-i\pi x} dx$

4) $FT(1) = \int_{-\infty}^{\infty} e^{-2\pi iux} dx = \delta(u)$

5) $FT(\cos(2\pi ux)) = \frac{1}{2} \left\{ \delta(u+m) + \delta(u-m) \right\}$

6) $FT[e^{-ax^2}](u) = \int_{-\infty}^{\infty} e^{-ax^2} e^{-2\pi iux} dx = \frac{\sqrt{\pi} e^{-\pi u^2/a}}{a}$
The optical art correlator.

In optical correlation we use the "4f" spatial filter arrangement to make a yes/no decision on a part. Yes, the part looks like the one to be determined or no it does not resemble the object to be determined. Let's use an example of your uncle's nose.

A bright spot at 2f indicates that yes, we are looking for a transparency of your uncle's nose. We can't make a commercial product detecting your uncle's nose, but, if we substitute a transparency of a virus, we can optically verify a virus in real time.
Matlab code listing

Bessel plot:

\[ x = \text{linspace}(-10, 10, 500); \quad \% \text{creates values for } x \]

\[ \text{plot}(x, \text{besselj}(0,x), 'c*') \quad \% \text{plots } j_0 \text{ bessel function as cyan asterisk} \]
\[ \text{hold on} \]
\[ \text{plot}(x, \text{besselj}(1,x), 'b-') \quad \% \text{plot } j_1 \text{ bessel function} \]
\[ \text{plot}(x, \text{besselj}(2,x), 'r+') \]
\[ \text{title('zero order in cyan, first order in blue, second order in red')} \]

Jinc or Bessel/x

\[ x = \text{linspace}(-20, 20, 2000); \quad \% \text{creates values for } x \]

\[ \text{plot}(x, (\text{besselj}(1,x))./x, 'c*') \quad \% ./ \text{for array division} \]
\[ \text{title('J1 Bessel function over } x \text{ or "Jinc" function')} \]

Plotting two Airy Disks

\[ x = \text{linspace}(-20, 20, 2000); \quad \% \text{creates values for } x \]

\[ \text{plot}(x, ((\text{besselj}(1,x))./x).^2, 'c*') \quad \% ./ \text{indicates array division} \]
\[ \text{hold on} \]
\[ \text{plot}(x, ((\text{besselj}(1,x-4))./(x-4)).^2, 'r*') \]
\[ \text{title('Airy disc or square of Jinc in cyan and Jinc displaced red')} \]

\[ \text{display('hit any key to continue')} \]
\[ \text{pause()} \quad \% \text{pause and wait for user to hit a key} \]
\[ \% \text{plot}(x, ((\text{besselj}(1,x-4))./(x-4)).^2 + (\text{besselj}(1,x)/x).^2, 'k+') \]
\[ \% \text{plot}(x, ((\text{besselj}(1,x*1.02))./(x*1.02)).^2 + (\text{besselj}(1,x)/x).^2, 'k+') \]
\[ \% \text{This plots the sum of the two area disks in black plus symbols} \]
\[ \text{hold off} \quad \% \text{This takes graphics hold off so new plots are in new window} \]